1. Let $x+y=15$ and let $y=x+9$. Find the value of $\frac{y}{x}$.

2. Find the largest possible value of the smaller of two different positive integers if the sum of these two positive integers must be 256.

3. Find the value of x such that $\dfrac{1}{2-\frac{1}{x}}=\dfrac{6}{11}$.

4. If $x^2+2.24x+c=0$, then the roots for x will be real numbers if $c \le k$. Find the smallest possible value of k. Express your answer as an exact decimal.

5. If x is a positive integer, find the sum of all distinct values of x such that $\sqrt[5]{x}<1.4$.

6. Assume non-zero denominators. If $\frac{594}{x}=\frac{132}{y}+\frac{297}{z}$, then $x=\frac{kyz}{wz+fy}$ where k, w, and f are positive integers. Find the smallest possible value of $(k+w+f)$.

7. Janie has 2 dimes and 2 nickels in a jar. She draws 2 coins without replacement at random from the jar. Find the probability that the 2 coins drawn have a total monetary value of 15 cents. Express your answer as a common fraction reduced to lowest terms.

8. Let $(144,92,136)$, (x,y,z), and (a,b,c) be three ordered triples of positive integers. No two respective members of any of these ordered triples differ by more than three. For example $|144-a| \le 3$. The sum of the members of each ordered triple is the same number. If $y=\frac{2}{3}c$, $b=\frac{2}{3}z$, and $x<a$, find the ordered triple (a,b,c). Express your answer as that ordered triple.

9. If 17 is added to nine times a certain positive number the result is 100 more than the square of the positive number that is 15 less than the original certain positive number. Find the original certain positive number.

10. Let a, b, and c be positive integers such that each is greater than 200. Let a have an odd number of integral divisors, let b have an odd number of integral divisors, and let c have exactly two integral divisors. If $a+b=c$, find the smallest possible value of c.

11. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

If $f(x)=mx+b$ and $f(7x)=7f(x+1)$, then:

A) The given information is contradictory and cannot exist.
B) $m=0$
C) $b=0$
D) $m=\frac{-6b}{7}$
E) $b+m=1$

Note: Be sure to write the correct capital letter for your answer.

12. A shop sells fudge for $5.50 per pound, and also sells caramel by the pound. One day the shop sold some fudge and also sold 4 pounds of caramel. The total sales of the fudge and caramel sold that day were $47.00, and the average price per pound was $4.70. Find the price of a pound of caramel. Express your answer in **dollars** and **cents**.

13. The solution set for x of the equation $\sqrt{kx-5}-\sqrt{3x-3}=\sqrt{px+2}$ is $(2,5)$. Find the value of k.

14. If $4x^2 + 32x = 5$ is expressed in the form $(x+a)^2 = b$, find the value of $(a+b)$. Express your answer as an improper fraction reduced to lowest terms.

15. Bob walks up an up-going escalator by stepping up 1 step at a time at a constant rate. It takes him 20 steps to the top. If he had stepped up 2 steps at a time, it would have taken him 12 steps to the top. Assuming he can step up 2 steps at a time just as fast as he can step up 1 step at a time, how many steps are there in the escalator?

16. A merchant purchased a TV set through his distributor. He marked up the price to 25% over his cost. Two weeks later, due to a need for cash, he was forced to sell the set at 37.5% under his cost. If the merchant lost $8 more per set than he would have made by selling at his original 25% markup, find the number of dollars that the TV set cost the merchant.

17. It is known that April 25, 2008 falls on a Friday. The next year after 2008 in which the month of February will contain five Fridays is the year x. Find the value of x.

18. If m is a real number, find the solution set for the equation: $m^2 = 3m$. Express your answer in **set form**.

19. If x represents a negative integer and if k represents a positive integer such that the largest value for x for which $|7x - k| > 82$ is -10, find the sum of all possible distinct values of k.

20. Given 3 consecutive odd integers represented by $x, y,$ and z (x, y, and z are not necessarily in consecutive order). If $x^3 + y^2 = z^3 + 1467$, find the sum of all possible distinct values for z.

AA
Algebra I

Name S08

School

______ Correct X 2 pts. ea. = [ ]

School Code

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 4
2. 127
3. 6
4. 1.2544 (must be this decimal)
5. 15
6. 31
7. $\frac{2}{3}$ (must be this reduced common fraction)
8. (145, 92, 135) (must be this ordered triple)
9. 28
10. 613
11. D
12. \$3.50 (\$ optional)
13. 16
14. $\frac{85}{4}$ (must be this reduced improper fraction)
15. 30 (steps optional)
16. 64 (\$ optional)
17. 2036
18. $\{0, 3\}$ or $\{3, 0\}$ (must be in set form)
19. 112
20. -2

ITEM ANALYSIS
149 Papers
% of correct
3AA Algebra I

1.	96%	11.	18%
2.	81%	12.	57%
3.	82%	13.	4%
4.	53%	14.	24%
5.	38%	15.	4%
6.	15%	16.	10%
7.	44%	17.	10%
8.	7%	18.	44%
9.	45%	19.	2%
10.	3%	20.	1%

173 Papers
% of correct
4AA Algebra I

1.	92%	11.	41%
2.	90%	12.	63%
3.	88%	13.	9%
4.	49%	14.	45%
5.	72%	15.	13%
6.	22%	16.	9%
7.	57%	17.	9%
8.	18%	18.	59%
9.	47%	19.	5%
10.	9%	20.	2%

1. Find the length of an edge of a cube whose volume is 50653.

2. **(Always, Sometimes, or Never)** For your answer, write the whole word **Always, Sometimes,** or **Never**—whichever is correct.

   If the diagonals of a quadrilateral are congruent and exactly one pair of sides of the quadrilateral is parallel, then the quadrilateral is an isosceles trapezoid.

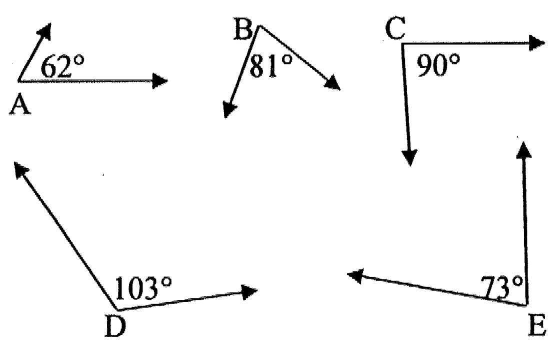
3. The circumference of the circle whose equation is $x^2+(y-3.4)^2=324$ can be expressed as $k\pi$. Find the value of k .

4. If x is an integer greater than 11, find the smallest value of x such that a triangle with sides of lengths 7, 11, and x is obtuse.

5. The perimeter of an original square is 63.6. A second square has an area that is 25% of the original square. Find the perimeter of the second square. Express your answer as a **decimal.**

6. The length of each slant height of a regular square pyramid is 11. The square base has a perimeter of 40. Find the total surface area of the pyramid.

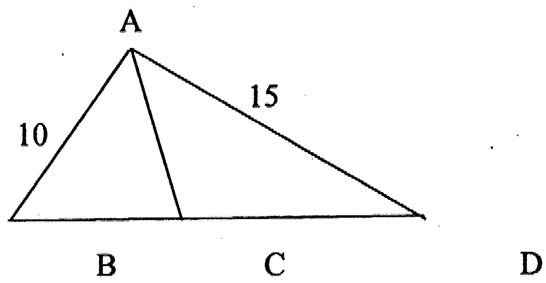
7. In the diagram there are five angles with degree measures as shown. If three of the five angles are selected (without replacement) at random, find the probability that exactly two are acute. Express your answer as a common fraction reduced to lowest terms.



8. The lengths of the diagonals of a rhombus are respectively 64 and 120. Find the length of a radius of the inscribed circle of the rhombus. Express your answer as an improper fraction reduced to lowest terms.

9. An inscribed angle of a circle intercepts an arc which is equal in degree measure to that of an interior angle of an equiangular pentagon. Find the degree measure of the inscribed angle.

10. In the diagram, B, C, and D are distinct collinear Points, and $\angle BAC \cong \angle DAC$. If $AB = 10$ and $AD = 15$, then the set of all possible values for AC is $\{AC : 0 < AC < k\}$. Find the value of k.



11. One endpoint of a diameter of the circle represented by $x^2 + 6x + y^2 = 14y - 21$ is $(3,8)$. Find the other endpoint. Express your answer as an **ordered pair** of the form (x, y).

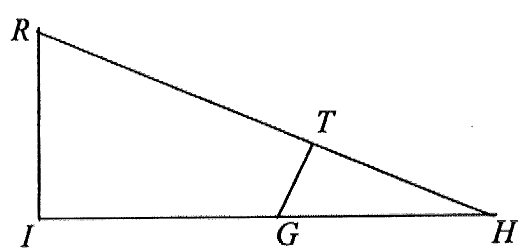
12. When is the first time after 1:00 that the minute hand and the hour hand will form an angle of $80°$? Express your answer in the form $h : m$ (hours:minutes).

13. Each leg of a right triangle has a length, which is an integer. Find the sum of the distinct perimeters of all such right triangles with a hypotenuse of 130.

14. **(Always, Sometimes, or Never)** For your answer, write the whole word **Always, Sometimes,** or **Never**—whichever is correct.

    If at least one pair of sides and at least one pair of angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

15. In the diagram (not necessarily drawn to scale), G lies in the interior of $\overline{HI}$, T lies in the interior of $\overline{RH}$, $\overline{GT} \perp \overline{RH}$, and $\overline{RI} \perp \overline{HI}$. $RI = 9$, $RT = 11$ and $HT = 4$. If ΔHGT is revolved one complete revolution around its longer leg, the volume of the solid formed by the revolution is $k\pi$. Find the value of k.



16. **(Always, Sometimes, or Never)** For your answer, write the whole word **Always, Sometimes,** or **Never**—whichever is correct.

    If a parallelogram is inscribed in a circle, then the length of at least one of the diagonals of the parallelogram is exactly twice the length of a radius of the circle.

17. Let $A_1, A_n, A_{(n+1)}$ be the vertices of a triangle in which $\overline{A_1A_n} \cong \overline{A_1A_{(n+1)}}$. Let $\angle A_{(n+1)}A_1A_n = 12°$. Let k be a positive integer such that $A_2, A_4, A_6, A_8, \ldots, A_{2k}$ in that order are points that lie between A_1 and A_n with A_2 closer to A_1, and such that $A_3, A_5, A_7, A_9, \ldots, A_{(2k+1)}$ in that order are points that lie between A_1 and $A_{(n+1)}$ with A_3 closer to A_1. Let p be a positive integer such that for all integral values of p from 1 to $(n-1)$ inclusive, isosceles triangles $A_pA_{(p+1)}A_{(p+2)}$ are formed such that in each triangle $\overline{A_pA_{(p+1)}} \cong \overline{A_{(p+1)}A_{(p+2)}}$. Find the value of n.

18. **(Multiple Choice)** For your answer write the **capital letter** which corresponds to the correct choice.

The geometric representation of the intersection of $x-3y=10$ and $2x-20=6y$ is:

A) A line
B) A point,
C) A ray
D) Two points
E) 0 points

Note: Be certain to write the correct capital letter as your answer.

19. Let O be the center of a circle in which points $A(10,5)$ and $D(-2,-1)$ lie on diameter $\overline{AB}$. Let $C(x, x+12)$ lie on a line that is tangent to the circle at B. If D is $\frac{3}{4}$ of the way from A to B, then the distance from C to O can be expressed in simplest radical form as $k\sqrt{w}$. Find the value of $(k+w)$.

20. A triangle has sides of lengths 20, 24, and 26. A circle is circumscribed about this triangle. **Rounded to the nearest integer**, find the area of the largest triangle that can be inscribed in this circle.

AA Geometry

Name S08

School S08

____ Correct X 2 pts. ea. = [ ]

School Code ____

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 37
2. Always
3. 36
4. 14
5. 31.8 (must be this decimal)
6. 320
7. $\frac{3}{5}$ (must be this reduced common fraction)
8. $\frac{480}{17}$ (must be this reduced improper fraction)
9. 54 (degrees optional)
10. 12
11. (−9, 6) (must be this ordered pair)
12. 1:20 (must be in hours:minutes form)
13. 1208
14. Sometimes
15. 12
16. Always
17. 8
18. A
19. 10
20. 243

ITEM ANALYSIS

159 Papers
% of correct
3AA Geometry

1.	95%	11.	41%
2.	87%	12.	51%
3.	56%	13.	18%
4.	66%	14.	79%
5.	71%	15.	25%
6.	53%	16.	84%
7.	26%	17.	3%
8.	27%	18.	79%
9.	64%	19.	9%
10.	8%	20.	6%

169 Papers
% of correct
4AA Geometry

1.	97%	11.	52%
2.	89%	12.	58%
3.	85%	13.	20%
4.	79%	14.	82%
5.	88%	15.	42%
6.	64%	16.	88%
7.	40%	17.	6%
8.	39%	18.	89%
9.	80%	19.	17%
10.	15%	20.	15%

1. Let $i = \sqrt{-1}$. Then $i^{13} - i^{7} = ki$ where k is a real number. Find the value of k.

2. Let a and b be real numbers, and let $i = \sqrt{-1}$. If $a + bi + 3 - 4i$ is a **positive** real whole number, find the smallest possible value of $(a+b)$.

3. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

   The graph of the equation $\frac{x}{16} + \frac{y}{9} = 1$ is a(an):

   A) Line
   B) Ellipse
   C) Parabola
   D) Hyperbola
   E) Point

   Note: Be certain to write the correct capital letter as your answer.

4. If three arithmetic means are inserted between -10 and 18, find the largest of the three insertions.

5. If $\begin{bmatrix} 3 & 2 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 26 \\ 55 \end{bmatrix}$, find the value of $(x+y)$.

6. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

   If $2x$ and $3y$ are both positive and $2x \neq 3y$, then, in regard to $2x$ and $3y$, the arithmetic mean:

   A) Is always greater than the positive geometric mean.
   B) Is greater than the positive geometric mean only if neither x nor y is equal to 1.
   C) Is greater than the positive geometric mean only if $x > 3$ and $y > 2$.
   D) Is greater than the positive geometric mean only if $2x > 3y$.
   E) Is never greater than the positive geometric mean.

   Note: Be certain to write the correct capital letter as your answer.

7. A person writes down 5 different integers at random from the 25 integers from 1 to 25 inclusive. Each of the 25 integers is then called off one at a time in a random order. As soon as all 5 of the person's numbers have been called off, the person yells: "Bingo." Find the probability that the person will have yelled "Bingo" before the 18th number is called. Express your answer as a decimal rounded to the nearest ten-thousandth.

8. The sequence 1, 2, 3, 4, 6, 8, 10, 12, 14, 17, 20, 23, 26, 29, 33 is an example of a quinterkle sequence. In a quinterkle sequence, there is a constant difference between consecutive terms of the first four terms. To find the fifth term, the constant difference increases by one. This new constant difference then continues through the ninth term. To find the tenth term, the constant difference again increases by one over the new constant difference. This process continues in this manner with the constant difference changing at every subsequent fifth term. If the example sequence were continued so that it had 256 terms, find the 256th term.

9. A circle has the equation of $(x-7)^2+(y+1)^2=10$. A line is tangent to the circle at the point $(10,-2)$. This tangent line passes through the point $(0,y)$. Find the value of y.

10. Let a, b, and c be three integers such that $f(x)=ax^5+bx^3+cx-29$ and such that $a+b+c<100$. If $f(2)=-39$ and $f(3)=91$, find the largest possible value of $(a+b+c)$.

11. An airplane whose capacity is 200 passengers is to be chartered for a flight to Europe. The fare is to be \$240 per person if 110 people buy tickets. However, the airline has agreed to reduce the fare for every passenger by \$1 for each additional ticket sold. Find the number of tickets that should be sold to maximize the total revenue for this chartered flight to Europe.

12. Linda plans to invest \$10,000 in a savings account that pays an annual percentage rate of 7.2% and is compounded monthly. After the interest is credited at the end of 4 months, find the value of Linda's investment. Round your answer to the nearest dollar, and express your answer as that whole number.

13. Let $a(10b+c)+d=fg^2+hg+e$. Let there be a one-to-one correspondence between the members of $\{a,b,c,d,e,f,g,h\}$ and the members of $\{2,3,4,5,6,7,8,9\}$. Find the value of $\left(fg^2+hg+e\right)$.

14. How many liters of distilled water should be added to a liter of a 40% acid solution to dilute it to a 30% acid solution? Express your answer as a common fraction reduced to lowest terms.

15. Let two real-numbered functions be defined as follows: $f(x)=4-x^2$ and $g(x)=1+\sqrt{x}$. The domain of the composite function $g \circ f$, when written in interval form, is $[k,w]$. Find the value of $(w-k)$.

16. A cubic polynomial in terms of x has zeroes of -2, 1, and 4. If the coefficient of x^3 is one, find the numerical coefficient of x.

17. A bag contains exactly 3 marbles—1 red, 1 white, and 1 blue. A girl draws a marble at random, replaces the marble, and continues to draw in this fashion. Find the probability that after 6 draws she has drawn exactly 2 marbles of each color. Express your answer as a common fraction reduced to lowest terms.

18. Let $i=\sqrt{-1}$. In an arithmetic sequence of complex numbers, the first term is $3+7i$, and the second term is $4+5i$. If the ninth term is written in the form $x+yi$ where x and y are real numbers, find the value of $(x-y)$.

19. A box contains 210 chips, each of which is either scarlet, gray, blue, or orange. There is at least one chip of each color. The number of gray chips is 5 times the number of scarlet chips. The number of blue chips is greater than three times the number of gray chips, and the number of orange chips is more than four times the number of gray chips. If the number of orange chips is more than the number of blue chips, find the smallest possible number of orange chips.

20. From 28 people—7 seniors, 7 juniors, 7 sophomores, and 7 ninth graders—a committee of five is appointed under the following conditions:

   a. There must be at least 1 senior;
   b. There cannot be more than 3 seniors;
   c. There cannot be more than 2 from any one of the non-senior groups;
   d. There must be at least one person from at least 2 non-senior groups.

Under these conditions, how many distinct committees are possible?

AA
Algebra II

Name S08

School

______ Correct X 2 pts. ea. = [ ]

School Code ______

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 2
2. 2
3. A
4. 11
5. 9
6. A
7. 0.1165 or .1165 (must be this decimal)
8. 6733
9. -32
10. 88
11. 175
12. 10,242 ($ optional)
13. 180
14. $\frac{1}{3}$ (must be this reduced common fraction) (liters optional)
15. 4
16. -6
17. $\frac{10}{81}$ (must be this reduced common fraction)
18. 20
19. 94 (chips optional)
20. 61740

ITEM ANALYSIS

154 Papers
% of correct
3AA Algebra II

1.	91%	11.	51%
2.	62%	12.	32%
3.	69%	13.	0%
4.	41%	14.	49%
5.	78%	15.	58%
6.	28%	16.	77%
7.	2%	17.	5%
8.	6%	18.	66%
9.	40%	19.	8%
10.	7%	20.	1%

173 Papers
% of correct
4AA Algebra II

1.	96%	11.	65%
2.	72%	12.	43%
3.	66%	13.	1%
4.	71%	14.	66%
5.	58%	15.	79%
6.	39%	16.	83%
7.	7%	17.	14%
8.	11%	18.	74%
9.	56%	19.	12%
10.	16%	20.	7%

1. Let $i=\sqrt{-1}$. Let k be a positive integer such that $24<k<28$. If $\left(4i^k\right)^3=64i$, find the value of k.

2. **(Multiple Choice) For your answer write the capital letter that corresponds to the best answer.**

   In any triangle ABC in which the degree measure of $\angle B$ is larger than the degree measure of $\angle C$, $\cos\left(90-\angle B+\angle C\right)^\circ=$

   A) $\sin\left(\angle B+\angle C\right)^\circ$
   B) $\sec\left(\angle B+\angle C\right)^\circ$
   C) $\sin\left(\angle B-\angle C\right)^\circ$
   D) $\sec\left(\angle B-\angle C\right)^\circ$
   E) $\cos\left(\angle B+\angle C\right)^\circ$

   Note: Be certain to write the correct capital letter as your answer.

3. Is the infinite series $1+2+4+8+\cdots+2^{(n-1)}+\cdots$ a convergent series or a divergent series? For your answer, write **convergent** or **divergent**, whichever is correct.

4. In ΔABC, $AB=10$, $BC=15$, and $AC=18$. Find the value of $\dfrac{\sin(\angle BAC)}{\sin(\angle BCA)}$

5. The lengths of the sides of a triangle are 36, 77, and 85. Find the measure of the smallest angle of the triangle. Give your answer in terms of degrees and minutes, rounded to the nearest minute.

6. Find the number of years it will take for a sum of money to triple if invested at an annual percentage rate of 5% and compounded continuously. Express your answer as a decimal rounded to the nearest hundredth of a year.

7. A spherically shaped balloon is being inflated so that the radius r is changing at the constant rate of 6 inches per second. If $r=0$ when the number of elapsed seconds is zero, then the number of cubic inches in the volume of the balloon after 5 seconds is $k\pi$. Find the value of k.

8. A bag contains exactly 6 marbles—3 red, 2 white, and 1 blue. A boy draws a marble at random, replaces the marble, and continues to draw in this fashion. Find the probability that his first 3 draws were marbles of the same color and that his final 2 draws were also marbles of the same color, but a different color from the first 3 marbles. Express your answer as a common fraction reduced to lowest terms.

9. Rounded to the nearest whole number of pounds, what force is necessary to pull a 7000-pound truck up a 5° incline?

10. If $t_n = (t_{(n-1)})^2 + t_{(n-1)}$ and if $t_1 = 1.1$, find the value of t_6. Round your answer to the nearest integer and express your answer as that rounded integer.

11. If $-40 \le x \le 70$ and $-70 \le y \le 50$, find the probability that a point (x, y) selected at random is in the interior of $\frac{(x-1)^2}{1089} + \frac{(y+14)^2}{3136} = 1$. Express your answer as a decimal rounded to the nearest ten-thousandth.

12. If $\vec{i} = (1,0)$ and $\vec{j} = (0,1)$, the two-dimensional vector whose length is 10 and which has the same direction as $3\vec{i} + 4\vec{j}$ is $k\vec{i} + w\vec{j}$. Find the ordered pair (k, w).

13. Let k be a positive integer. When $(a_1 + a_2 + a_3 + a_4 + \cdots + a_k)^9$ is expanded and completely simplified, the number of terms is 2505433700. Find the value of k.

14. In this problem, assume that the standard deviation is calculated according to the standard method of calculating the standard deviation for a set of sample proportions. Also, assume the following table of z-scores with the accompanying standard normal probabilities is accurate. On the table $-2(.0228)$ means that there is a normal probability of .0228 of obtaining a z-score of -2 or less.

−2(.0228)	−1.9(.0287)	−1.8(.0359)	−1.7(.0446)	−1.6(.0548)
−1.5(.0668)	−1.45(.0735)	−1.4(.0808)	−1.35(.0885)	−1.3(.0968)
−1.25(.1056)	−1.2(.1151)	−1.1(.1357)	−1(.1587)	−0.9(.1841)
−0.8(.2119)	−0.75(.2266)	−0.7(.2420)	−0.6(.2743)	−0.5(.3085)
−0.4(.3446)	−0.3(.3821)	−0.25(.4013)	−0.1(.4602)	0(.5000)
0.1(.5398)	0.2(.5793)	0.25(.5987)	0.3(.6179)	0.4(.6554)
0.5(.6915)	0.6(.7257)	0.7(.7580)	0.75(.7734)	0.8(.7881)
0.9(.8159)	1.0(.8413)	1.1(.8643)	1.2(.8849)	1.25(.8944)
1.3(.9032)	1.4(.9192)	1.5(.9332)	1.6(.9452)	1.7(.9554)
1.75(.9599)	1.8(.9641)	1.9(.9713)	2.0(.9772)	2.1(.9821)
2.2(.9861)	2.25(.9878)	2.3(.9893)	2.4(.9918)	2.5(.9938)

Assume that 64% of all mathletes like chocolate milk shakes. In a sample observation, Karen selects 400 mathletes at random. Find the probability that the proportion of those mathletes who like chocolate milk shakes is between 61% and 70%. Express your answer as a **decimal** rounded to the nearest thousandth.

15. An apartment rental company has 400 apartments available, and 260 are presently rented at \$1000 per month. The company has decided it will only raise or lower the rent per month by integral increments of \$50 applied to *all* rental units.. A survey has shown that, so long as there are apartments available, for each \$50 drop in rent per month, there will be 30 new tenants. Find the number of dollars in the monthly rent that will maximize total income.

16. Find the limit of the sequence: $4,-2,1,-\frac{1}{2},\frac{1}{4},\cdots,(-1)^{(n+1)}(\frac{1}{2})^{(n-3)},\cdots$

17. $\overline{AB}$ is a line segment with a length of 10. D is a point on $\overline{AB}$ that is located somewhere at random within a length of 3 from B. A second point, C, is located somewhere at random on $\overline{AB}$. If it is known that D is four times as likely to be within 2 to 3 units of B as it is to be 0 to 2 units of B, find the probability that D is closer to C than C is to A. Express your answer as a common fraction reduced to lowest terms.

18. Find the ordered pair of real numbers that satisfy the equation $x^2+y^2+5-4y+2x=0$. Be sure to express your answer as an **ordered pair** of the form (x,y).

19. Let $a_n=\dfrac{3n^2+5n^3+7n+9}{n-3}$. Let $L=\lim\limits_{n\to 0}(\dfrac{3n^2+5n^3+7n+9}{n-3})$. If n is a positive integer such that $0<n<22$, find the sum of all distinct n such that a_n-L is a positive integer.

20. Maglio hits a waist high (3.247 feet) fastball with an initial velocity of 112.8 feet per second at an angle of elevation with the horizontal of 42.48° and directly into a headwind of a 5.987 feet per second wind. Assume the effect-of-gravity vector is $(0,-16t^2)$ where t is measured in seconds. If the ground is flat, find the number of feet in the maximum height above the ground that the ball will reach. Express your answer as a decimal rounded to the nearest hundredth.

AA
Pre-Calculus

Name S08

School ______

______ Correct X 2 pts. ea. = [ ]

School Code ______

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 27
2. C
3. divergent
4. 1.5 or $1\frac{1}{2}$ or $\frac{3}{2}$
5. 25°3' (must be in degrees and minutes form)
6. 21.97 (must be this decimal) (years optional)
7. 36,000
8. $\frac{19}{648}$ (must be this reduced common fraction)
9. 610 (kilograms optional)
10. 19,686,983
11. 0.4398 or .4398 (must be this decimal)
12. (6, 8) (must be this ordered pair)
13. 42
14. 0.888 or .888 (must be this decimal)
15. 800 ($ optional) or 700
16. 0
17. $\frac{61}{100}$ (must be this reduced common fraction)
18. (–1, 2) (must be this ordered pair)
19. 76
20. 93.92 (must be this decimal) or 84.55

ITEM ANALYSIS

168 Papers
% of correct
3AA Pre-Calculu

1.	60%	11.	17%
2.	71%	12.	64%
3.	81%	13.	2%
4.	73%	14.	5%
5.	54%	15.	52%
6.	58%	16.	58%
7.	54%	17.	4%
8.	24%	18.	54%
9.	35%	19.	14%
10.	66%	20.	14%

175 Papers
% of correct
4AA Pre-Calculu

1.	61%	11.	33%
2.	75%	12.	82%
3.	92%	13.	9%
4.	87%	14.	11%
5.	69%	15.	58%
6.	60%	16.	68%
7.	68%	17.	9%
8.	37%	18.	68%
9.	49%	19.	30%
10.	81%	20.	27%

1. Find the perimeter of a square whose diagonal has a length of $\sqrt{50}$.

2. **(Always, Sometimes, or Never)** For your answer, write the whole word **Always, Sometimes,** or **Never**—whichever is correct.

   If the average of three different positive integers is 2, then one of those positive integers is 2.

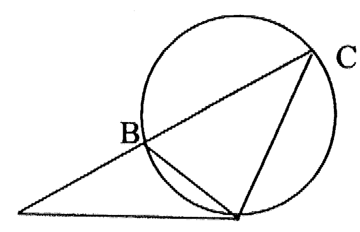
3. Four line segments have lengths as follows: $AB = 5$, $CD = 8$, $EF = 10$, and $GH = 16$. If three of these line segments are selected at random without replacement, find the probability that these three segments can be placed on a plane surface in such a manner as to be the sides of a triangle. Express your answer as a common fraction reduced to lowest terms.

4. Katie has only nickels, dimes, and quarters in her purse. She has 1 more dime than quarters and twice as many nickels as dimes. If the value of the coins in Katie's purse is $2.45, find the number of **dimes** that Katie has in her purse.

5. **(Always, Sometimes, or Never)** For your answer, write the whole word **Always, Sometimes,** or **Never**—whichever is correct.

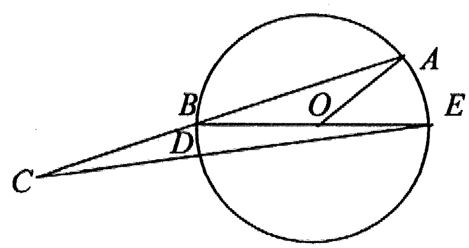
   If none of the angles of a kite is a right angle, then the vertices of the kite are concyclic.

6. In the diagram, points A, B, and C are collinear, points B, C, and D lie on the circle, and point D is a point of tangency of the tangent segment from A to D. If $AB = 16$, $BC = 9$, and $CD = 10$, find BD.

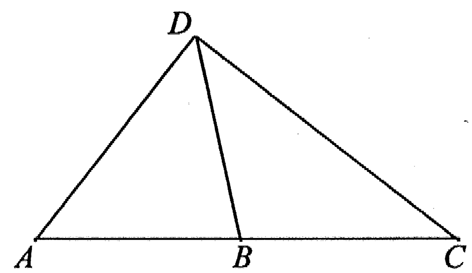


7. A polynomial $p(x)$ has integral coefficients. When $p(x)$ is divided by $x-2$, the remainder is 19. When $p(x)$ is divided by -5, the remainder is 34. The remainder when $p(x)$ is divided by $x^2-7x+10$ is $kx+w$ where k and w are positive integers. Find the value of $(k+w)$.

8. Points A, B, D, and E lie on the circle. Point O, the center of the circle lies on diameter $\overline{BE}$. Point B lies on $\overline{AC}$, and point D lies on $\overline{CE}$. If $\angle AOB = 104°$ and $\angle BEC = 30°$, find the degree measure of $\angle ACE$.



9. In the diagram, $AB = 5$, and $BC = 6$. If A, B, and C are collinear, the ratio of the area of ΔABD to the area of ΔCBD can be expressed as $\frac{k}{w}$ where k and w are positive integers. Find the smallest possible value of $(2k+3w)$.



10. $\sqrt{124+70\sqrt{3}} = a+b\sqrt{c}$ where a, b, and c are positive integers. Find the value of $(a+b+c)$.

11. One of the digits of a two-digit number is 9, and the other digit is non-zero. If the two-digit number is an integral multiple of 3, find the sum of all distinct possibilities for the two-digit number.

12. The length of one leg of a right triangle is 26, and the length of the radius of the inscribed circle of the right triangle is 8. Find the length of the hypotenuse. Express your answer as an improper fraction reduced to lowest terms.

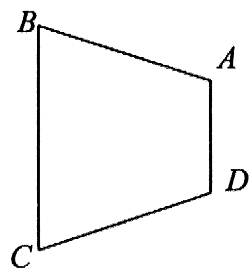
13. If the expression $x^2+4-y^2+4x-9-6y$ were factored over the integers into $(x+ay+b)(x-cy-d)$ where a, b, c, and d are positive integers, find the value of $(a+b+c+d)$.

NO CALCULATORS

14. In the long run, Liz gets to class ahead of Britany $\frac{2}{3}$ of the time. Find the probability that Liz got to class ahead of Britany at least twice out of three specified class periods. Express your answer as a common fraction reduced to lowest terms.

15. If $x^2 - 4x + 6\sqrt{2} - 5 = 0$, then the sum of the squares of the roots for x can be expressed, in simplest radical form, as $k - w\sqrt{f}$. Find the value of $(k + w + f)$.

16. In the diagram, $ABCD$ is an isosceles trapezoid with $\overline{BC}$ as one of the bases. $AD = x+1$, $DC = x+4$, $BC = 3x-1$, and $AB = 2x$. Find the value of the perimeter of $ABCD$.



17. Given the 4 points: $A(2,18)$, $B(48,26)$, $C(20,50)$, and $D(30,40)$. Find the area of the region that is in the exterior of ΔACD but is in the interior of ΔABC.

18. If x is a real number such that $x^4 - 1 = 80$, find the smallest possible value of $x^3 + 2x^2 + 2x - 1$.

19. Let a and b be distinct digits such that the four-digit number with respective digits from left to right of $abba$ is the product of 4 consecutive positive odd integers. Find the sum of these 4 consecutive positive odd integers.

20. One 12-hour clock is running at a constant gain of 6 minutes per 24 hours. A second 12-hour clock is running at a constant loss of 7 minutes per 8 hours. The first clock now shows a time of 12:00, and the second clock now shows a time of 1:30. When these two clocks show identical times the next time from now, the second clock is immediately adjusted so that it then runs at a constant loss of 1 minute per 8 hours. What time will be showing on these two clocks when these two clocks next show identical times after the second clock is adjusted? Express your answer in the form $h:m$ (**hours: minutes**). **Note:** "identical times" means, for example, if one clock shows 9:20, then the other clock will also show 9:20.

AA
Fr/So 8 Person

School ____S08____

____ Correct X 5 pts. ea. = [ ]

School Code ________

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

1. 20
2. Always
3. $\frac{1}{2}$ (must be this reduced common fraction)
4. 6
5. Never
6. 8
7. 14 ** THIS PROBLEM WAS NOT COUNTED
8. 8 (degrees optional)
9. 28
10. 15
11. 396
12. $\frac{194}{5}$ (must be this reduced improper fraction)
13. 8
14. $\frac{20}{27}$ (must be this reduced common fraction)
15. 40
16. 32
17. 414
18. – 16
19. 40
20. 4:20 (must be in hours:minutes form)

ITEM ANALYSIS

29 Papers
% of correct
3AA 9-10 8-Per.

1.	90%	11.	76%
2.	66%	12.	10%
3.	72%	13.	38%
4.	93%	14.	10%
5.	59%	15.	10%
6.	24%	16.	97%
7.	**	17.	3%
8.	62%	18.	69%
9.	76%	19.	45%
10.	14%	20.	0%

37 Papers
% of correct
4AA 9-10 8-Per.

1.	95%	11.	73%
2.	89%	12.	32%
3.	62%	13.	59%
4.	95%	14.	22%
5.	62%	15.	32%
6.	24%	16.	95%
7.	**	17.	3%
8.	78%	18.	84%
9.	68%	19.	65%
10.	22%	20.	3%

NO CALCULATORS

1. The largest negative value of x such that $|x-13| \geq 51$ is k. Find the value of k.

2. $\begin{vmatrix} 1 & 2 & 4 \\ 3 & 5 & 6 \\ -2 & -1 & -5 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 4 \\ 14 & 12 & k \\ -2 & -1 & -5 \end{vmatrix}$. Find the value of k.

3. If $\log_3 7 + \log_3 x = 4$, find the value of x. Express your answer as an improper fraction reduced to lowest terms.

4. If x is an integer, find the sum of all distinct x such that $|x+5| + |x| = 5$.

5. In Takeummath High School, 40% of the students are sophomores, 32% of the students are juniors, and the rest are seniors. It is known that 98% of the sophomores are enrolled in a math course, 90% of the juniors are enrolled in a math course, and 50% of the seniors are enrolled in a math course. If a student is selected at random from Takeummath High School, find the probability that the student selected is enrolled in a math course. Express your answer as a **decimal,** not as a percentage.

6. If $i = \sqrt{-1}$, then $\dfrac{(3-i)(7+2i)}{3-4i} = a+bi$ where a and b are real numbers. Find the value of $(a+b)$. Express your answer as a **decimal**.

7. Let k and w represent positive integers such that $\log_2 6 - \log_2 x + \log_2 3 = \log_2(\dfrac{k}{wx})$. Find the smallest possible value of $(k-w)$.

8. $ABCD$ is a square. $\overleftrightarrow{AJ}$ intersects $\overline{BC}$. Perpendiculars to $\overleftrightarrow{AJ}$ from B, C, and D meet $\overleftrightarrow{AJ}$ in F, G, and E respectively. If $DE = 24$, and $BF = 12$, find AG.

NO CALCULATORS

NO CALCULATORS

NO CALCULATORS

9. A position vector is defined as a vector whose initial point is at the origin that is at the point $(0,0)$. If the position vector $(-5,2)$ is translated so that its new initial point is $(3,1)$, find its new terminal point. Express your answer as an **ordered pair**.

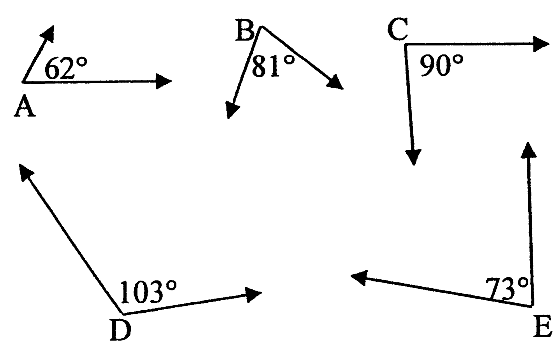
10. April 30, 2005 fell on a Saturday. Find the next time that April 30 in the year x falls on a Saturday, that April 30 in the year y falls on a Saturday, and that $y-x=12$. For your answer, give the value of $(x+y)$.

11. Let a, b, and c be **single digit** positive integers such that $2a+b+c+112=148$. Find the value of $(3a+2b+c)$.

12. Two coplanar congruent circles are externally tangent at Point A. Chord $\overline{AB}$ of the first circle is $\perp$ to chord $\overline{AC}$ of the second circle. If $\angle ACB = 15°$, and $AB = 6$, then the area of the first circle is $\pi(k+p\sqrt{w})$ where $p\sqrt{w}$ is in simplest radical form. Find the value of $(k+p+w)$.

13. Let $A=\{x+2y, 3x+5, 2x+19, x+y+10\}$. The arithmetic mean of the 4 members of A is 32. If x and y are both positive integers, how many distinct ordered pairs of the form (x,y) exist?

14. In the diagram there are five angles with degree measures as shown. If three of the five angles are selected (with replacement) at random, find the probability that exactly two are acute. Express your answer as a common fraction reduced to lowest terms.



NO CALCULATORS

15. A line is neither horizontal nor vertical. One of the intercepts of the line is three times the second intercept of the line, but only one of the intercepts is an integer. If the point $(56,126)$ is on the line, find the value of the x-intercept. **Note: the x-intercept is the value of x when the line intersects the x-axis, and is not an ordered pair. Similarly, the y-intercept is the value of y when the line intersects the y-axis, and is not an ordered pair.**

16. Find the third term of an arithmetic sequence if the ninth term is 3 and the seventeenth term is 27.

17. Set $A=\{1,2,3,4,5,6,7,8,9,10\}$. Set $B=\{4,5,6,7\}$. Bob and Judy are playing a game. Bob draws a number at random from Set A, replaces the number, and then again draws a number at random from Set A. Judy draws a number at random from Set B. Bob wins if at least one of his drawn numbers is greater than the number that Judy drew. Otherwise, Judy wins. Find the probability that Bob wins. Express your answer as a common fraction reduced to lowest terms.

18. If $(3x-2y)^7$ is expanded and completely simplified, then one of the terms in the expansion is kx^5y^2. Find the value of k.

19. The two lines represented by $3x-5y=k$ and $wx-37y=-4694$ meet at a point whose y-coordinate is 2. If the tangent of the positive acute angle formed by the lines is $\frac{21}{20}$ and if $w>0$, find the value of $(k+w)$.

20. Find the two ordered triples of numbers (x,y,z) that satisfy $\begin{cases} xy+3x+3y=46 \\ xz+3x+3z=101 \\ yz+3y+3z=41 \end{cases}$.

Express your answer as **ordered triples** of the form (x,y,z).

AA
Jr/Sr 8 Person

School ______ S08 ______

______ Correct X 5 pts. ea. = [ ]

School Code ______________

Note: **All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.**

1. −38

2. 35

3. $\frac{81}{7}$ (must be this reduced improper fraction)

4. −15

5. 0.82 or .82 (must be this decimal)

6. 6.48 (must be this decimal)

7. 17

8. 36

9. (−2, 3) (must be this ordered pair)

10. 4190

11. 54

12. 111

13. 5

14. $\frac{54}{125}$ (must be this reduced common fraction)

15. 434

16. −15

17. $\frac{137}{200}$ (must be this reduced common fraction)

18. 20412

19. 71

20. (8, 2, 7), (-14, -8, -13) (must have two ordered triples in either order)

ITEM ANALYSIS

26 Papers
% of correct
3AA 11-12 8-Per.

1.	84%	11.	92%
2.	36%	12.	8%
3.	84%	13.	52%
4.	76%	14.	16%
5.	92%	15.	8%
6.	60%	16.	88%
7.	76%	17.	20%
8.	36%	18.	64%
9.	84%	19.	0%
10.	0%	20.	8%

38 Papers
% of correct
4AA 11-12 8-Per.

1.	95%	11.	95%
2.	53%	12.	5%
3.	97%	13.	74%
4.	63%	14.	24%
5.	84%	15.	26%
6.	68%	16.	87%
7.	89%	17.	32%
8.	42%	18.	55%
9.	89%	19.	0%
10.	0%	20.	18%

1. If 6.234 is a root for x of $5.67x^2 + kx - 712.14 = 0$, find the value of k.

2. In this problem, assume that the standard deviation is calculated according to the standard method of calculating the standard deviation for a set of sample proportions. Also, assume the following table of z-scores with the accompanying standard normal probabilities is accurate. On the table 0.1(.5398) means that there is a normal probability of .5398 of obtaining a z-score of 0.1 or less.

−0.8(.2119)	−0.75(.2266)	−0.7(.2420)	−0.6(.2743)	−0.5(.3085)
−0.4(.3446)	−0.3(.3821)	−0.25(.4013)	−0.1(.4602)	0(.5000)
0.1(.5398)	0.2(.5793)	0.25(.5987)	0.3(.6179)	0.4(.6554)
0.5(.6915)	0.6(.7257)	0.7(.7580)	0.75(.7734)	0.8(.7881)
0.9(.8159)	1.0(.8413)	1.1(.8643)	1.2(.8849)	1.25(.8944)
1.3(.9032)	1.4(.9192)	1.5(.9332)	1.6(.9452)	1.7(.9554)
1.75(.9599)	1.8(.9641)	1.9(.9713)	2.0(.9772)	2.1(.9821)

In a survey of 10,000 citizens, the arithmetic mean for the amount of federal income tax paid per citizen was \$12,000 with a standard deviation of \$2,000. Assume that the results for the amount of federal income tax paid per citizen in this survey were normally distributed. If three of the 10,000 citizens were selected at random, find the probability that at least two of the three paid income tax within the range of \$11,000 to \$16,000. Express your answer as a **decimal** rounded to the **nearest hundredth.** Do **not** use scientific notation.

3. The equation of the line passing through the points $(-4.868, 12.89)$ and $(19.12, 56.13)$ can be expressed in the form $y = mx + b$. Find the value of $(m + b)$.

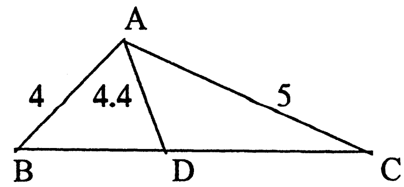
4. Tom rows at a constant speed of 3 mph. Kay rows at a constant speed of 6.3478 mph. The speed of a two-person boat with both people rowing is 75% of the sum of their separate rowing speeds. Find the number of **seconds** it will take Tom and Kay to row 100 yards in a two person boat.

5. If $f(x) = \log(x+3)$ and $g(x) = \cos(2x)$, find the value of $f(g(4))$.

6. Find the positive real solution to $x^{x^{x^{x}}} = 100$.

7. A rectangular floor is 19.348 feet by 15.647 feet. At 43 cents per square foot, find the cost of painting this floor. Express your answer in dollars and cents with your answer rounded to the nearest cent. Do **not** use scientific notation.

8. In ΔABC, $AB=4$, $AD=4.4$, and $AC=5$. If $\angle BAD \cong \angle CAD$, find BC. Express your answer as a **decimal** rounded to the nearest tenth. **Do not use scientific notation.**



9. Point O is the center of the circular lower base of a right circular cylinder, and point P is the center of the circular upper base of this cylinder. $\overline{BA} \parallel \overline{PO}$ and $\overline{BA}$ is a lateral edge of the cylinder. $OB=14$, and $\angle OBA=30°$. Find the total surface area of the cylinder.

10. Nuk L. Hedd invested \$5000 in a bank on January 3, 1998, in a 10 year Certificate of Deposit at an annual percentage rate of 9.419% interest compounded continuously. Nuk L. claimed the bank told him on January 3, 2008, that after the interest was credited that his Certificate of Deposit, rounded to the nearest dollar, was worth \$12,809. Cal Kew took out his calculator, punched a few buttons and said: "Nuk L., rounded to the nearest dollar, your bank was off by k dollars." Assuming Cal is correct, find the value of k. Express your answer as an **integer**.

11. In ΔABC, $AB=76.78$, $BC=83.42$, and the number of degrees in the measure of $\angle ABC$ is an integral multiple of 16. Find the largest possible area of ΔABC.

12. In ΔABC with $A(7,9)$, $B(-8,-3)$, and $C(11,2)$, D is a point on $\overline{BC}$ such that $\frac{BD}{DC}=\frac{1}{2}$. F is a point on $\overline{AC}$ such that $\frac{CF}{FA}=\frac{1}{3}$. G is a point on $\overline{AD}$ such that $\frac{AG}{GD}=\frac{1}{4}$. H is a point on $\overline{AF}$ such that $\frac{AH}{HF}=\frac{1}{5}$. Find the area of ΔGHF.

13. If x is a radian measure, how many distinct solutions to $\cos(2x)=2\cos(x)$ exist in $[-10\pi, 10\pi]$? Express your answer as an **integer**.

14. If r represents the length of a radius of a circle, find the sum of all distinct r such that $r < 4$ and such that the area of the circle is an integer. Round your answer for that sum to the nearest integer and express your answer as that integer.

15. If $f(x) = 1.6x(1-x)$, find the value of $f^{\infty}(0.2)$ where f^{∞} represents the limit of an infinite sequence of repeated iterations using 0.2 as the initial value of x. Express your answer as a **decimal** rounded to the nearest thousandth. Do **not** use scientific notation.

16. The original price of a radio was k. A manager marked up the original price by 50% to a new price. After 7 days, the manager reduced the new price by 50% to obtain a price of \$105.45. Find the value of k. Express your answer as the **exact amount** in **dollars** and **cents.** Do **not** use scientific notation.

17. The polar coordinates (r,θ) of the vertices of ΔABC and of point D are $A(5,30°)$, $B(7,315°)$, $C(8,100°)$, and $D(3,57°)$. Find the absolute value of the shortest distance possible from D to $\overleftrightarrow{AC}$.

18. $$\begin{cases} x+y=77.23 \\ y+z=145.68 \\ x+z=97.28 \end{cases}$$

Find the **exact** value of $(3x+3y+3z)$, and express your answer as a **decimal.** Do **not** use scientific notation.

19. A steroid test correctly predicts presence of steroids 98.12% of the time, and correctly identifies absence of steroids 95.14% of the time. A group of NCAA athletes was tested, and 15.08% of them tested positive for steroid use. Find the fractional part of athletes that actually used steroids. Express your answer as a **decimal.** Do **not** use scientific notation or percentages.

20. Points A, B, C, D, E, and F lie on a circle with center at O. From an external point P in the plane of the circle, secant segments $\overline{PC}$, $\overline{PB}$, and $\overline{PD}$ are drawn, such that E is between P and C, F is between P and D, and such that P, A, O, and B are collinear in that order. If $\overset{\frown}{BD} = 62°$, $\overset{\frown}{BC} = 78°$, and $\overset{\frown}{AE} = 12°$, find the degree measure of $\angle BPD$. Express your answer as a **decimal** in degrees rounded to the nearest hundredth of a degree. Do **not** use scientific notation.

AA
Calculator Team

School S08

_____ Correct X 5 pts. ea. = [ ]

School Code ____________

Note: **All answers must be written legibly. Round answers to four significant digits and write in either standard or scientific notation unless otherwise specified in the question. Except where noted, angles are in radians. No units of measurement are required.**

1. 78.89 or 7.889×10 or 7.889×10^1
2. 0.74 or .74 (must be this decimal)
3. 23.47 or 2.347×10 or 2.347×10^1
4. 29.18 or 2.918×10 or 2.918×10^1
5. 0.4555 or .4555 or 4.555×10^{-1}
6. 1.866 or 1.866×10^0
7. 130.18 (must be this decimal) ($ optional)
8. 1.6 (must be this decimal)
9. 841.1 or 8.411×10^2
10. 15 (must be this integer) ($ optional)
11. 3185 or 3.185×10^3
12. 6.375 or 6.375×10^0
13. 20 (must be this integer)
14. 135 (must be this integer)
15. 0.375 or .375 (must be this decimal)
16. 140.60 (must be this decimal) ($ optional)
17. 1.835 or 1.835×10^0
18. 480.285 (must be this decimal)
19. 0.1096 or .1096 (must be this decimal)
20. 26.54 (must be this decimal) (degrees optional)

ITEM ANALYSIS

31 Papers
% of correct
3AA Calculator

1.	61%	11.	35%
2.	3%	12.	0%
3.	81%	13.	55%
4.	42%	14.	16%
5.	61%	15.	32%
6.	39%	16.	94%
7.	81%	17.	13%
8.	0%	18.	61%
9.	26%	19.	6%
10.	68%	20.	0%

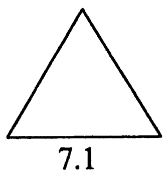
34 Papers
% of correct
4AA Calculator

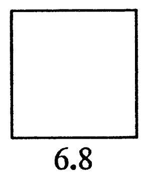
1.	62%	11.	44%
2.	21%	12.	3%
3.	79%	13.	74%
4.	56%	14.	32%
5.	56%	15.	53%
6.	68%	16.	94%
7.	74%	17.	24%
8.	12%	18.	62%
9.	53%	19.	12%
10.	91%	20.	6%

1. The length of the longest chord of a circle whose equation is $(x+14)^2+y^2=23.5225$ is k. Find the value of k. Express your answer as an **exact decimal.**

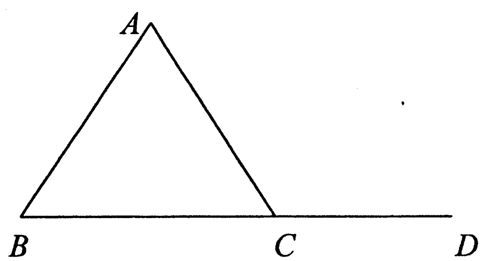
2. The slope of the line that passes through $(\sqrt{2},\sqrt{3})$ and $\left(\sqrt{18},k\sqrt{w}\right)$ is $\frac{3\sqrt{6}}{4}$. If k and w are positive integers, find the smallest possible value of $(k+w)$.

3. Given the equilateral triangle and the square with lengths as shown. Find the sum of the areas of the inscribed circles of each figure. Express your answer as a **decimal** rounded to the nearest hundredth.





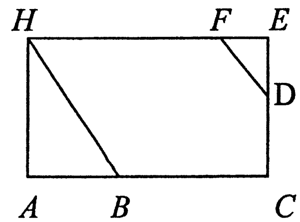
4. Points B, C, and D are collinear. $\angle BAC=(x-6)°$, $\angle ACD=(x+70)°$, $\angle ABC=(2x-16)°$, and $\angle ACB=k°$. $f(x)=\frac{x^2+bx+c}{x+d}$ has zeroes when $x=8$ and when $x=11$ and is undefined when $x=2$. Find the value of $(k+b+c+d)$.



5. Let (x,y) be the point of reflection of $(10,10)$ with respect to the line whose equation is $y=2x$. Let p be the perimeter of an equilateral triangle whose area is $\sqrt{432}$. Find the value of $(2x+3y+p)$.

6. An equiangular convex polygon contains 48 sides. Find the degree measure of an interior angle of the polygon. Express your answer as a decimal.

7. $ACEH$ is a rectangle with $\angle AHB \cong \angle FDE$. A, B, and C are collinear, C, D, and E are collinear, and E, F, and H are collinear. $EC = 10$, $DC = 5$, $AB = 8$.
Find: $(EF + HB)$.



8. Johnny Miller had one penny, one nickel, one dime, one quarter, and one half-dollar. If Johnny chooses exactly three of those five coins, find the sum total of all the different sums of the monetary value of the three coins chosen. Express your answer in **cents**, and **not** in dollars and cents.

9. A 5 by 5 by 5 cube is painted red and then cut into 1 by 1 by 1 cubes. Let k be the number of distinct smaller cubes that are painted red on exactly 2 sides. Let w be the number of sides of a convex polygon whose interior angles have a total measure of $2520°$. Find the value of $(k + w)$.

10. Let $A = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$. How many different subsets of A consist only of distinct members whose sum is 15?

ANSWERS

1. 9.7 (must be this decimal)

2. 7

3. 49.51 (must be this decimal)

4. 131

5. $46 + 12\sqrt{3}$ or $12\sqrt{3} + 46$

6. 172.5 (must be this decimal) (degrees optional)

7. $4 + 2\sqrt{41}$ or $2\sqrt{41} + 4$

8. 546 (cents optional)

9. 52

10. 26

11. Let k be the largest of three consecutive **odd** integers if the product of the smallest and largest is 160 more than 37 times the middle integer. Let w be the length of the shortest altitude of the triangle whose sides have lengths of 52, 73, and 75. Find the value of $(k+w)$.

12. Let $A=\{1,2,3,5\}$ and let $B=\{8,11,15,17,19,23\}$. Two distinct members of A are the roots for x of the quadratic equation $x^2+bx+c=0$, and two distinct members of B are the roots for x of the quadratic equation $x^2+kx+w=0$. Find the largest possible value for $(b+w)$.

13. In a circle, two chords intersect in the interior of the circle. One chord is divided into two segments of respective lengths of 29.1, and 48.6. The other chord is divided into two segments of respective lengths 19.4 and k. Find the value of k. Express your answer as a decimal.

14. If the system $\begin{cases} \frac{1}{x}-\frac{1}{y}=-\frac{1}{2} \\ x=3y-1 \end{cases}$ is solved, one of the ordered pairs in the solution consists of two positive integers. Find that ordered pair. Express your answer as an **ordered pair** of the form (x,y).

15. A line passes through the point $(-9,0)$ and is parallel to the line whose equation is $2x-5y+7=0$. The equation of the line can be expressed in the form $y=mx+b$. Find the value of $(11m+20b)$. Express your answer as a **decimal.**

ANSWERS

11. 91

12. 434

13. 72.9 (must be this decimal)

14. (2, 1) (must be this ordered pair)

15. 76.4 (must be this decimal)

AA

Fr/So 2 Person Team

School S08

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	Score (to be filled in by proctor)
1. 9.7 (must be this decimal)	
2. 7	
3. 49.51 (must be this decimal)	
4. 131	
5. $46+12\sqrt{3}$ or $12\sqrt{3}+46$	
6. 172.5 (must be this decimal) (degrees optional)	
7. $4+2\sqrt{41}$ or $2\sqrt{41}+4$	
8. 546 (cents optional)	
9. 52	
10. 26	

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. 91
12. 434
13. 72.9 (must be this decimal)
14. $(2,1)$ (must be this ordered pair)
15. 76.4 (must be this decimal)

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

PLUS: 2 point bonus for being first In round with correct answer

1. Find the total number of distinct non-real roots for the following equation:

$$8x^{15}-7x^{14}-6x^{12}-5x^{10}-4x^{8}-3x^{6}-2x^{4}-x^{2}-621=0.$$

2. If the following system is solved, one of the ordered pairs (x,y) in the solution set has a non-integral value for y. Find that value of y. Express your answer as a **decimal.**

$$\begin{cases} x^2+2xy=21 \\ y=2x-16 \end{cases}$$

3. Let k be the number of terminal zeroes in which $89!$ ends. If $f(x)=x^2+\frac{4}{x^3}$ and $g(x)=\log(x)+\frac{3}{x^2}$, let w be the value of $f(g(3.5))$. Find the value of $(k+w)$. Express your answer as a **decimal** rounded to the nearest tenth.

4. An arithmetic sequence has a 40th term of 300 and a 20th term of 100. Let x be the value of the 50th term of this arithmetic sequence. Let y be the sum of the infinite geometric sequence whose 1st term is 12 and whose 2nd term is 9. Find the value of $(x+y)$.

5. Let k be the sum of all distinct positive integers m such that $\log_2 m<6$. Let w be the number of distinct negative values in the sequence:
$\tan(37°), \tan(137°), \tan(237°), \sin(37°), \sin(137°), \sin(237°), \cos(37°), \cos(137°), \cos(237°)$
Find the value of $(k+w)$.

6. Let $2x+3y=54$. Let k be the sum of all distinct values of x such that both x and y are positive integers. Let w be the length of the longest possible side of a triangle that has an angle of 120° and two sides of lengths 5 and 16. Find the value of $(k+w)$.

7. If $x>0$, then the set of all x such that $7x^3+81x^2+188x-96\geq 0$ is $\{x : x\geq k\}$. Find the value of k. Express your answer as a common fraction reduced to lowest terms.

8. The rational root theorem is used to make a list of all possible rational roots for the equation $9x^4-27x^3-94x^2+12x+40=0$. From this list, a number is selected at random and is found to be a non-integer. Find the probability that the chosen number is a root of the equation.

9. Let p be the perimeter of a **right** triangle whose area is 486 and which has an angle whose cosecant is 1.25. Let k be the smallest positive integer that has exactly 28 distinct positive integral divisors. Find the value of $(p+k)$.

10. If x, y, and z are positive integers, how many distinct ordered triples (x,y,z) exist such that $x+y+z=6$?

ANSWERS

1. 14
2. – 17.2 (must be this decimal)
3. 28.8 (must be this decimal)
4. 448
5. 2020
6. 127
7. $\frac{3}{7}$ (must be this reduced common fraction)
8. $\frac{1}{16}$ or 0.0625 or .0625 or 6.25%
9. 1068
10. 10

11. The parametric equations of a curve are $x = 5\cos(t)$ and $y = \sin(t)$. The Cartesian equation of this curve is $x^2 + ky^2 = w$. Find the value of $(k+w)$.

12. Find the value of $\sum_{a=1}^{50}(3a) + \sum_{k=5}^{37}(k+9)$.

13. The equation $x^5 + ax^2 - bx^3 + cx + d = 0$ has roots for x of 1, 2, 3, 4, and k. Let w be an integer such that the roots for y of the equation $y^3 - 15y^2 + wy + 120 = 0$ form an arithmetic sequence. Find the value of $(k+w)$.

14. Point $A(-33,56)$ lies on the terminal ray of an angle in standard position. Find the sine of the angle. Express your answer as a common fraction reduced to lowest terms.

15. If θ is an angle measured in radians, find the period, in radians, of the graph of the equation $y = -2\tan(-\frac{1}{4}\theta)$.

ANSWERS

11. 50

12. 4815

13. 16

14. $\frac{56}{65}$ (must be this reduced common fraction)

15. 4π

AA

Jr/Sr 2 Person Team

School S08

(Use full school name – no abbreviations)

Total Score (see below*) = [ ]

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

Answer	**Score** (to be filled in by proctor)
1. 14	______
2. -17.2 (must be this decimal)	______
3. 28.8 (must be this decimal)	______
4. 448	______
5. 2020	______
6. 127	______
7. $3/7$ (must be this reduced common fraction)	______
8. $1/16$ or 0.0625 or .0625 or 6.25%	______
9. 1068	______
10. 10	______

TOTAL SCORE: ______

(*enter in box above)

Extra Questions:

11. 50
12. 4815
13. 16
14. $56/65$ (must be this reduced common fraction)
15. 4π

*** Scoring rules:**

Correct in 1st minute – 6 points

Correct in 2nd minute – 4 points

Correct in 3rd minute – 3 points

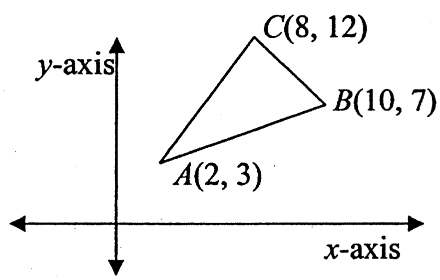
PLUS: 2 point bonus for being first In round with correct answer

1. Let k be the common difference or the common ratio of the sequence: $\log_4 3, \log_4 6, \log_4 12, \log_4 24, \log_4 48$. Let w be the sum of the infinite geometric sequence whose first two terms are respectively 5 and $\frac{65}{23}$. Find the value of $(k+w)$.

2. If a, b, and c are **single digit** positive integers, how many distinct ordered triples exist such that $3a+4b+c=65$?

3. If $g(x)=\frac{x^2-9}{x^2-16}$, then in interval notation, the **range** of values of g is $(-\infty,k] \cup (w,\infty)$. Find the value of $(k+w)$. Express your answer as an improper fraction reduced to lowest terms.

4. For both k and w, use the diagram with coordinates as shown. Let k be the length of the median from C to $\overline{AB}$. Let ΔDAB with $D(8,w)$ have an area of 140. If $w>31.3$, find the exact value of $(k+w)$.



5. Let k be the remainder when $(2x^{21}-3x^{16}+2)$ is divided by $(x+1)$. Let w be the largest integer value of x which makes $\sum_{n=0}^{\infty}\frac{(2x+7)^n}{3^{(n+1)}}$ a convergent series. Find the product (kw).

6. If $\log_4(m)=y+\log_2(n)$, then $m=k^y(n^w)$ where k and w are positive integers. Find the value of $(2k+3w)$.

7. Find the value of $\sin^2(1°)+\sin^2(2°)+\sin^2(3°)+\cdots+\sin^2(90°)$. Express your answer as an exact decimal.

8. A sequence is defined as follows: $a_n = (-1)^{(n+1)} 2^{(2n-1)}$. This same sequence can be defined recursively as follows: $a_1 = k,\ a_{(n+1)} = w(a_n)$ if $n \geq 1$. Find the value of $(k+w)$.

9. Let $k = \frac{w}{6}$ where w is a positive integer such that $w < 100$. Find the sum of all distinct values of k such that the roots for x of $2x^2 - 3kx + k = 0$ are not real. Express your answer as an improper fraction reduced to lowest terms.

10. Let n be a positive integer such that $\sum_{k=0}^{n}(k+1)^2 - \sum_{k=0}^{n}k^2 = 169$. ΔABC is a right triangle with $\angle ACB = 90°$, $AC = 4$ and $BC = \sqrt{48}$. Let
$w = \frac{(AC)(BC)}{2} - \frac{(BC)^2 \sin(\angle ABC)\sin(\angle ACB)}{2\sin(\angle CAB)}$. Find the value of $(n+w)$.

S AA 08

Jr/Sr 2 Person Team Demonstration Questions

School ____________________

School Code ____________________

Note: All answers must be written legibly in simplest form, according to the specifications stated in the Contest Manual. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required.

	Answer	Score (to be filled in by proctor)
1.	12	______
2.	5	______
3.	$25/16$ (must be this reduced improper fraction)	______
4.	$41+\sqrt{53}$ or $\sqrt{53}+41$	______
5.	9	______
6.	14	______
7.	45.5 (must be this decimal)	______
8.	-2	______
9.	$5/2$ (must be this reduced improper fraction)	______
10.	12	______

FRESHMAN-SOPHOMORE RELAY COMPETITION

ICTM 2008 DIVISION AA STATE FINALS ROUND 1

1. When x^2+9x-5 is multiplied by $3x^2+5x+4$ and the result is completely simplified, the coefficient of x^3 is k and the coefficient of x^2 is w. Find the value of $(2k+3w)$.

2. The sum of the two distinct roots for x of the equation $10.375x^2-(ANS)x-1992=0$ is k. Find the value of k.

3. Find the slope of the line that is perpendicular to the line whose equation is $2x+(ANS)y=12$.

4. A circle whose equation is $x^2-(ANS)x+y^2+12y+3=0$ has its center at (a,b) and has a radius whose length is r. Find the value of $(2a+3b+4r)$.

ANSWERS

1. 166

2. 16

3. 8

4. 18

FRESHMAN-SOPHOMORE RELAY COMPETITION

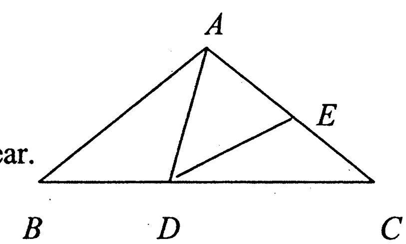
ICTM 2008 DIVISION AA STATE FINALS ROUND 2

1. If 6 is one of the roots for x of the quadratic equation $x^2 + kx + 18 = 0$, find the other root for x.

2. $(ANS + 3)$ consecutive integers are given in ascending order. The sum of the second, fourth, and fifth is 41. Find the sum of the third and the greatest of these $(ANS + 3)$ terms.

3. In a town designed by an eccentric geometer, there are only $(ANS - 19)$ roads. Each road is straight, but exactly two are parallel, and at every intersection exactly two of the roads meet. Find the number of intersections there are in this town. Assume that each road is long enough that it intersects every road possible under the given conditions.

4. In the diagram, $AB = AC$, $AE = AD$, $\angle BAD = (ANS)°$, points A, E, and C are collinear, and points B, D, and C are collinear. Find the degree measure of $\angle EDC$.



ANSWERS

1. 3

2. 29

3. 44

4. 22 (degrees optional)

FRESHMAN-SOPHOMORE RELAY COMPETITION

ICTM 2008 DIVISION AA STATE FINALS ROUND 3

1. On April 1, Matrix department store marked a prom dress up by 20%. This new April 1 price was not changed until May 1 of that year when this prom dress was marked down by 20% of its new April 1 price. There were no further price changes. If Hayley bought this prom dress on May 2 of that year for \$86.40 (before tax), find the number of dollars in the price (before tax) of this prom dress on March 31 of that year.

2. A triangular number is a number that can be written as the sum of consecutive positive integers beginning with 1. For example, 6 is a triangular number since $1+2+3=6$. Find the number of distinct triangular numbers that are greater than 19 and less than ANS.

3. If ANS people are at a meeting and each person shakes hands exactly once with every other person at the meeting, find the number of distinct handshakes that took place at this meeting.

4. Let x be the perimeter of a right triangle in which the lengths of all sides of the right triangle are integers. If $x<180$, find the sum of all possible distinct values of x if the length of one side of the right triangle is ANS.

ANSWERS

1. 90 (\$ optional)

2. 7

3. 21

4. 322

FRESHMAN-SOPHOMORE RELAY COMPETITION

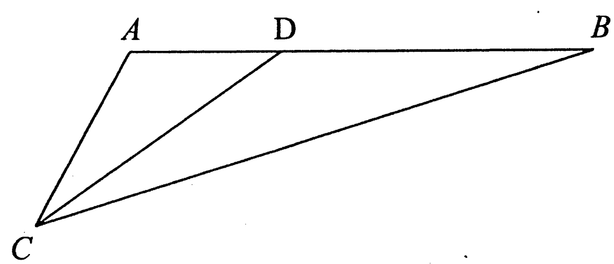
ICTM 2008 DIVISION AA STATE FINALS ROUND 4

1. Find the x-coordinate **only** of the solution to the system:

$$\begin{cases} \dfrac{4}{x}+\dfrac{3}{y}=4 \\ \dfrac{4}{x}-\dfrac{1}{y}=0 \end{cases}$$

2. Find the **positive** value of x such that $(x-2)(x+1)=ANS$.

3. In the diagram, points A, D, and B are collinear, $\angle ABC=32°$, $\overline{CD}\cong\overline{BD}$, and the degree measure of $\angle CAB$ is ANS times the degree measure of $\angle DCB$. Find the degree measure of $\angle ACD$.



4. A triangle has an area of 30 square units and a perimeter of ANS units. Find the length of a radius of the inscribed circle of the triangle.

ANSWERS

1. 4
2. 3
3. 20
4. 3

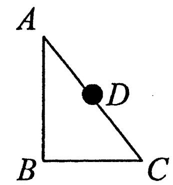
FRESHMAN-SOPHOMORE RELAY COMPETITION

ICTM 2008 DIVISION AA STATE FINALS ROUND 5

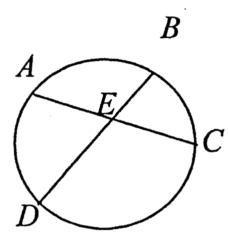
1. Find the slope of a line that passes through the points $(3,9)$ and $(-2,-6)$.

2. Find the value of $(x+y)$ by solving the system: $\begin{cases} x+3y=11 \\ 3x-6y=ANS \end{cases}$

3. In the diagram, $\angle ABC=90°$, $BC=ANS+1$, $AB=15$, and D is the midpoint of the hypotenuse. Find BD.



4. In the diagram chords $\overline{AC}$ and $\overline{BD}$ intersect at E. If $AE=ANS$, $CE=8$, and $DE=10$, find BD.



ANSWERS

1. 3

2. 7

3. $\frac{17}{2}$ or $8\frac{1}{2}$ or 8.5

4. $\frac{84}{5}$ or $16\frac{4}{5}$ or 16.8

JUNIOR-SENIOR RELAY COMPETITION

ICTM 2008 DIVISION AA STATE FINALS ROUND 1

1. Find the sum of the roots of the cubic equation $5x^2 = 7 + 2x^3$. Express your answer as a **decimal.**

2. Find the sum of the solutions of $\log_4(x) + \log_x(4) = ANS$.

3. Find the length of the major axis of the ellipse whose equation is: $9x^2 - 18x + 4y^2 + 16y + 7 = ANS$.

4. The lengths of the three sides of a triangle are ANS, 7, and 9. Rounded to the nearest minute, find the measure of the angle of the triangle that is opposite the side whose length is ANS. Express your answer in the form $k°w'$.

ANSWERS

1. 2.5 (must be this decimal)

2. 18

3. 6

4. 41°45' (must be in degrees and minutes form)

JUNIOR-SENIOR RELAY COMPETITION

ICTM 2008 DIVISION AA STATE FINALS ROUND 2

1. Solve the determinant equation for x: $\begin{vmatrix} 7 & 2-x \\ 5 & 3 \end{vmatrix} = 76$.

2. A parabola of the form $y = ax^2 + bx + c$ passes through the points $(-1, 40)$, $(2, ANS)$, and $(5, 4)$. The axis of symmetry of this parabola can be expressed in the form $x = k$. Find the value of k.

3. Find the number of distinct committees of ANS persons that can possibly be appointed from 12 persons.

4. The graph of $y = \dfrac{7x+3}{x - ANS}$ has a vertical asymptote of $x = k$ and a horizontal asymptote of $y = w$. Find the value of $(2k + 3w)$.

ANSWERS

1. 13
2. 5
3. 792
4. 1605

JUNIOR-SENIOR RELAY COMPETITION

ICTM 2008 DIVISION AA STATE FINALS ROUND 3

1. For all positive integers x and y, $f(x+y)=f(x)+f(y)$. If $f(1)=6$, find $f(10)$.

2. A parabola has the equation $y=x^2+4x+\frac{ANS}{4}$. Find the exact distance from the vertex of the parabola to the point at which the parabola intersects the y-axis.

3. The length of a radius of a circle is ANS. A quadrilateral inscribed in this circle is also circumscribed about a circle concentric with the first circle. Find the area of the quadrilateral.

4. $\sum_{x=0}^{x=k}\left(\log\left(y^{(2^x)}\right)\right)=p(\log(y))$. If $k=\left|\frac{ANS+8}{4}\right|$, find the value of p.

ANSWERS

1. 60

2. $2\sqrt{5}$ (Must be this simplified radical.)

3. 40

4. 8191

JUNIOR-SENIOR RELAY COMPETITION

ICTM 2008 DIVISION AA STATE FINALS ROUND 4

1. If the expression $(x+2)^4$ is expanded and completely simplified, find the numerical coefficient of x^3.

2. The first term of an arithmetic sequence is 3, and the last term is 357. If the arithmetic sequence contains ANS terms, find the sum of the terms in the arithmetic sequence.

3. In ΔABC, $\angle ACB = 20°$, $BC = 2000$, and $AC = ANS$. Rounded to the nearest **whole number**, find the length of $\overline{AB}$.

4. In ΔXYZ, $XY = ANS$, $YZ = 900$, and $\angle XYZ = 62°$. Rounded to the nearest **whole number**, find the area of ΔXYZ.

ANSWERS

1. 8

2. 1440

3. 813

4. 323,026

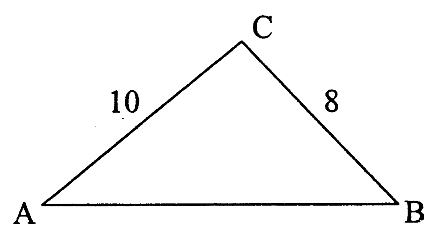
JUNIOR-SENIOR RELAY COMPETITION

ICTM 2008 DIVISION AA STATE FINALS ROUND 5

1. Let $(x+2y)^6$ be expanded and completely simplified. If the expansion is then written in terms of descending powers of x, find the sum of the digits of the numerical coefficient of the fourth term in this expansion.

2. Let $n = ANS$. Find the n^{th} term of the arithmetic sequence whose first term is 5 and whose fifth term is 10. Express your answer as a **decimal**.

3. In the diagram with lengths as shown, the degree measure of $\angle BAC$ is $(2(ANS)+5)$. Find the value of $\sin(\angle ABC)$. If the answer is rational, express your answer as a common fraction reduced to lowest terms. If the answer is irrational, express your answer in simplest radical form.



4. ANS should be in the form $\frac{a}{b}$ or $a\sqrt{b}$. Let $n=(a+b)$. Find the value of x such that $8(4^n) = 16^{(3x+5)}$.

ANSWERS

1. 7

2. 12.5 (must be this decimal)

3. $\frac{5}{8}$ (must be this reduced common fraction)

4. $\frac{3}{4}$ or 0.75 or .75

2008 ICTM State Mathematics Contest
Oral Competition--Voting Theory

1. An important first step in creating a mathematical model is making explicit the assumptions of the model. What are the assumptions of the spatial model of election competition?

2. A recent survey asked 1200 voters to position themselves on a 7-point scale regarding their feelings about government health insurance. A voter placing himself or herself at 1 feels there should be a government insurance plan which would cover all medical and hospital expenses for everyone. A voter placing himself or herself at 7 feels that medical expenses should be paid by individuals, and through private insurance plans like Blue Cross. Placing the responses along a continuum from 0 to 1, we have the following distribution:

Public Opinion on Government Health Insurance

Position	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Location on [0, 1]	**0.1**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.9**
% of Voters	**21**	**12**	**13**	**20**	**14**	**11**	**9**

a. Assuming these survey results are representative of the electorate as a whole, what is the expected position of a voter drawn randomly from the set of all voters?

b. Assume the election is a two-candidate election and candidate A adopts the expected position of the randomly chosen voter. What position can candidate B choose in order to maximize his or her vote total? Is this position unique? If not, what is the range of positions B can choose?

c. Assume that the liberal candidate A positions himself or herself at 0.4 and the conservative candidate B positions himself or herself at 0.6. If a third candidate C enters the race, what position should candidate C adopt? What will be the final vote totals?

3. Suppose the following preference lists represent the true preferences of the 101 voters involved in an election to be decided by the approval method.

	Number of Voters		
Rank	**61**	**20**	**20**
First	A	B	C
Second	B	A	B
Third	C	C	A

a. If each voter approves of his two top choices, who will win the election?

b. Does this example illustrate a violation of the Condorcet winner criterion? Explain.

2008 ICTM State Mathematics Oral Competition
Extemporaneous Questions--Voting Theory

1. As of February 24 political pundits were saying that Hillary Clinton needed to win 2 of the 3 upcoming big state primaries in order to remain a viable candidate. The three big states were Ohio, Texas, and Pennsylvania. Recent polls had the voters intending to vote in the Democratic primaries stating their preferences as follows:

Ohio		**Texas**		**Pennsylvania**	
Hillary Clinton	54%	Hillary Clinton	43%	Hillary Clinton	56%
Barack Obama	46%	Barack Obama	57%	Barack Obama	44%

As of February 24, what was the probability that Hillary Clinton would win at least 2 of these 3 big state primaries?

2. Describe the electoral college as a weighted voting system. Include a description of the pivotal voter.

3. What are some inadequacies of the spatial model of election competitions?

2008 ICTM State Mathematics Contest
Voting Theory--Judge's Information Sheet

1. Assumptions of the spatial model of election competition:

1) Candidates seek to maximize the total number of votes they receive.
2) Voter decisions are based entirely on the positions candidates take on some single overriding issue.
3) Positions of candidates and voters on this issue can be represented as points on a left-right continuum, ranging from very liberal on the left to very conservative on the right.
4) Candidates know the distribution of voters' positions.
5) Voters vote for the candidate whose position along the continuum is closest to their own.
6) All voters vote.
7) Both voters and candidates are rational, i.e., they both choose the course of action that best satisfies their goals.

2. a. The expected position of a voter drawn randomly from the set of all voters is the mean $\overline{I}$ of the distribution:

$$\overline{I}=\left(\frac{1}{100}\right)\left[21(0.1)+12(0.3)+13(0.4)+20(0.5)+14(0.6)+11(0.7)+9(0.9)\right]=.451$$

b. If B takes a position just to the right of A, say at .46, then B's vote total will be 54% and A's will be 46%. This position is not unique. B's vote total will be 54% with any position greater than .451 and less than .549.

c. C can maximize his or her vote total by assuming a position just to the left of A, say at 0.38. (C cannot win the election.) Since A and B are equidistant from 0.5, they will split the votes at 0.5. The final vote totals will be 44% for B, 33% for C, and 23% for A.

3. a. The vote totals are: A - 81, B - 101, C - 20. Thus, B wins.

b. Yes, this example shows that the approval method of deciding elections can violate the Condorcet winner criterion. Although A is a Condorcet winner, the approval method chooses B as the winner of the election.

2008 ICTM State Mathematics Oral Competition
Extemporaneous Questions--Voting Theory
Judges' Information Sheet

1. Hillary Clinton wins at least 2 of the 3 big state primaries if she wins exactly 2 states (and loses the third) or if she wins all three states. The probability that this occurs is $(.54)(.43)(.44) + (.54)(.56)(.57) + (.43)(.56)(.46) + (.54)(.43)(.56) = .515$.

2. The electoral college can be thought of as a weighted voting system with 51 voters--the 50 states plus the District of Columbia. The weight of each vote is the sum of the number of Representatives and the number of Senators for the given state. The weight of the vote for the District of Columbia is 3--the same as the weight of a small state with only one Representative. Since there are 435 Representatives in the House of Representatives and 100 Senators in the Senate, the sum of the weighted votes is 435 + 100 + 3, or 538. The quota is a simple majority of the 538 votes, or 270 votes. As the election returns come in state by state, the pivotal voter is the state whose electoral votes put the candidate's total over the quota (270). (In the infamous 2000 Presidential election, the pivotal voter was Florida.)

3. There are several ways in which the spatial model is not realistic:

1) There is not usually just one overriding issue that determines voter choice.
2) Voter choice is generally affected by many factors besides the candidates' position on policy--the voter's party affiliation, the personality, integrity, competence, and leadership ability of the candidate, sociodemographic characteristics, current economic conditions, response to incumbent performance, etc.
3) While the model posits that voters will vote for the candidate whose position on policy is nearest their own, in practice voters know that a) the candidate may be adopting the position just to win votes and may not be sincere and b) regardless of candidate intention, he or she will probably not be able to fully implement the policy they are advocating once elected. Thus, while the candidate's stated position may be closest to the voter's own view, the voter may decide not to vote for that candidate.
4) The model assumes that all voters will vote for the candidate whose position is closest to their own. However, the closest position may still be so far away that the voter sees the candidate as not representing his or her point of view. The voter may be so disaffected that he or she stays home and does not vote at all.