Competition A – Pre-Calculus – Individual Contest

Team Make-up: maximum 6 students; any grade level
Questions: 20
Time: 50 minutes
Format: Individual competition
All battery operated calculators permitted, including CAS-type
Answers must be legible
Answers must be **exact** unless otherwise indicated in the question
Scoring: Correct answers worth 2 points each; 40 points possible per team member
Team Score: The top 4 scores count toward the team score; 5th and 6th scores are used for tie-breaking
Maximum 160 points per team

Sample Regional Questions

1. Let $k = \sin\left(\frac{2}{3}\pi\right)$ and $w = \cot\left(\frac{5}{6}\pi\right)$. Determine the exact product (kw).

   Answer: $-\frac{3}{2}$ or $\frac{-3}{2}$ or -1.5 or $-1\frac{1}{2}$

2. The area between the curves $r = 6\cos\theta$ and $r = 2\cos\theta$ is $k\pi$. Determine the value of k.

   Answer: 8

3. Let $f(x) = (x-a)(x-b)^2(x+c)$ with $0 < a < b < c$. Determine the solution for $f(x) < 0$. Express your answer(s) in interval notation. (For example, the interval (k, w) represents $k < x < w$.)

   Answer: $(-c, a)$

4. In a geometric sequence, the last term is 1458, the common ratio is -3, and the sum of the terms is 1094. Determine the second term of this geometric sequence.

   Answer: –6

5. Determine the numerical coefficient of the sixth term when $(k+2w)^8$ is expanded and written in order of decreasing powers of k.

   Answer: 1792

6. A bag contains red and blue marbles. When two marbles are drawn, the probability that they are both red is equal to the probability they are both blue. The probability that one of each color is drawn is $\frac{4}{7}$. Find the total number of marbles in the bag.

Answer: 8

7. $\angle A$ and $\angle B$ are first quadrant angles such that $\sin A = \frac{4}{7}$ and $\cos B = \frac{5}{7}$. In reduced and simplified radical form, $\tan(A+B) = \frac{k\sqrt{p} + w\sqrt{q}}{f}$ where k, w, p, q, and f are integers. Determine the value of the sum $(k+w+p+q+f)$.

Answer: 62

8. Points A and B lie on circle O. The radius of circle O is 12. $AB = 10$. Determine the length of $\overset{\frown}{AB}$. Express your answer as a decimal rounded to the nearest tenth.

Answer: 10.3

9. Find the product of the solutions for the equation $x^3 - 7x^2 - 14x + 67 = 0$.

Answer: –67

10. The equation of the directrix of a parabola with vertex $(-3, 2)$ that passes through $(9, -16)$ and opens down is either $x = k$ or $y = w$. Determine the equation of this directrix. Express your answer as the equation $x = k$ or $y = w$.

Answer: y = 4