Competition B – Junior-Senior 8 Person Team

Team Make-up: maximum 8 students, no more than 4 may be seniors
Questions: 20
Time: 20 minutes
Format: Team members work together and submit one answer sheet
NO Calculators permitted
Answers must be legible
Answers must be **exact** unless otherwise indicated in the question
Scoring: Correct answers are worth 5 points each; Maximum 100 points possible per team

Sample Regional Questions

1. Let $\log_2 5 = x$ and $\log_2 9 = y$. Then $\log_2 243000 = ax + by + c$. Determine the sum $(a+b+c)$. Express your answer as a decimal.

   Answer: 8.5

2. Let $k = \lim_{x \to 16} \left(\frac{\sqrt{x} - 4}{x - 16} \right)$. Determine the value of k. Express your answer as an integer or as a common or improper fraction.

   Answer: $\frac{1}{8}$

3. Determine the smallest positive composite integer that has exactly 21 distinct positive integral factors. Express your answer as that single simplified integer. ("Composite" means the product of two or more prime powers.)

   Answer: 576

4. The graph of $y = \frac{x^2 + 3x + 1}{x - 1}$ has one or more asymptotes whose equations are of the forms $x = a$, $y = b$, and/or $y = kx + w$. Determine the sum of all values of a, b, k, and w that exist for this graph.

   Answer: 6

5. All angles are measured in radians. Determine the sum $\sin(0) + \sin\left(\frac{\pi}{4}\right) + \sin 2\left(\frac{\pi}{4}\right) + \sin 3\left(\frac{\pi}{4}\right) + \cdots + \sin 2016\left(\frac{\pi}{4}\right)$.

   Answer: 0

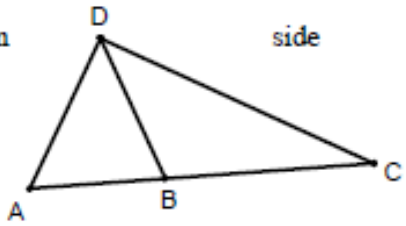
6. Let $x^2+\frac{1}{x^2}=3$. Then the largest real solution for x can be written as $\frac{k+w\sqrt{p}}{q}$ in reduced and simplified radical form with k, w, p, and q integers with $q>0$. Determine the value of the sum $(k+w+p+q)$.

Answer: 9

7. $k=\log_2\left(\log_9\left(\log_2\left(\log_2 256\right)\right)\right)$. Determine the exact value of k.

Answer: -1

8. ΔACD (not necessarily drawn to scale) is shown with point B on side $\overline{AC}$. $\cos\angle DBC=-\frac{3}{4}$, $BD=6$, and $AD=10$. Determine the exact length AB.



Answer: $\frac{9+\sqrt{337}}{2}$

9. The line $y=mx+b$ is represented by the parametric equations $x=4t-1$ and $y=3t+2$. Find the value of m. Write your answer as a common or improper fraction reduced to lowest terms.

Answer: $\frac{3}{4}$

10. When a polynomial $P(x)$ of degree $n>2$ is divided by $x-3$, the remainder is 5. When $P(x)$ is divided by $x-1$, the remainder is 1. Find the remainder when $P(x)$ is divided by x^2-4x+3.

Answer: $2x-1$ or $-1+2x$